INTERNATIONAL WORKSHOP ON

DIFFERENTIAL EQUATIONS

ON THE OCCASION OF
LUIS SANCHEZ'S 70TH BIRTHDAY

5-6
SEPTEMBER
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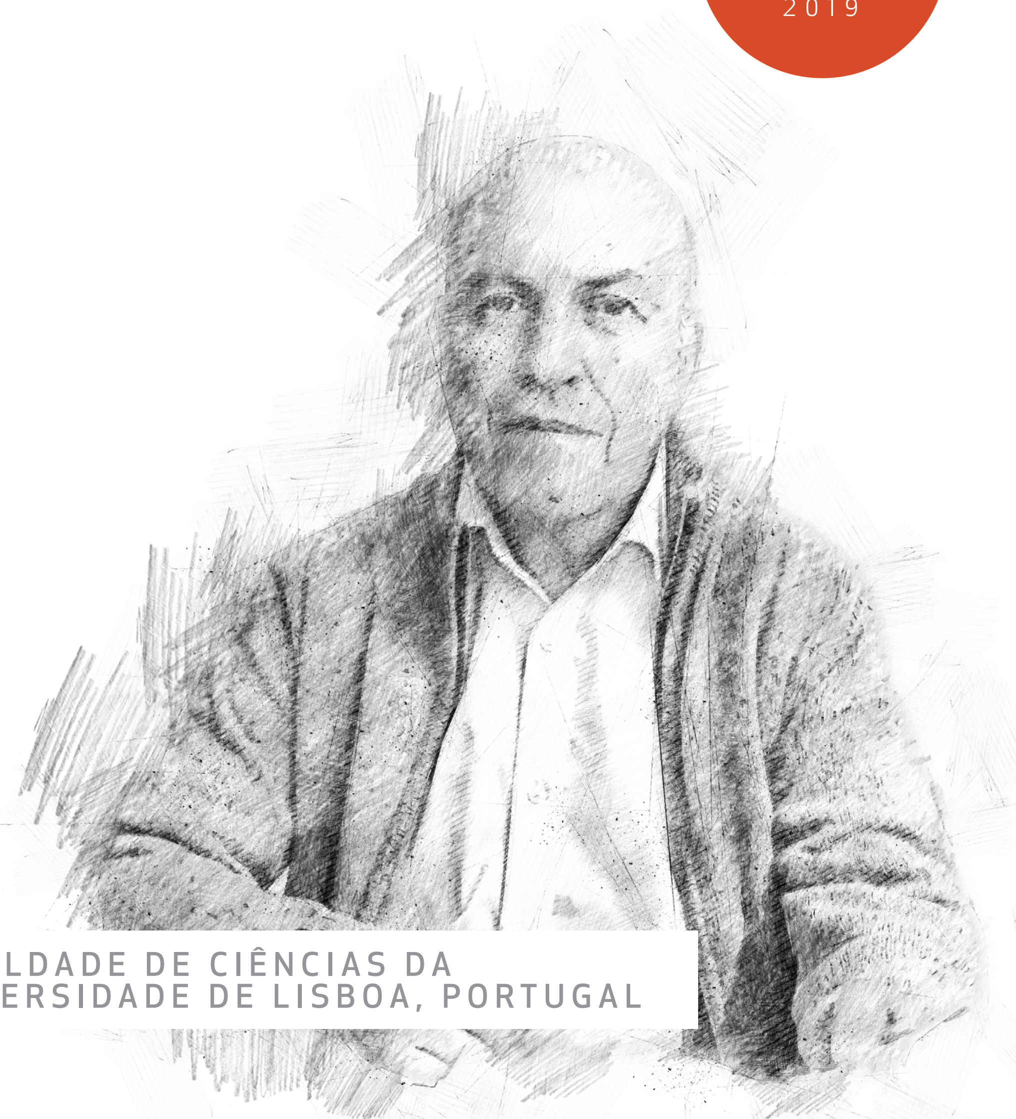

FACULDADE DE CIÊNCIAS DA
UNIVERSIDADE DE LISBOA, PORTUGAL

International Workshop

on

Differential Equations

On the Occasion of Luís Sanchez's 70th Birthday

Book of Abstracts

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Faculdade de Ciências da Universidade de Lisboa

Portugal

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Foreword

On the Occasion of Luís Sanchez's 70th Birthday

The International Workshop of Differential Equations (*Diffsanchez 70*) is designed to be a celebration in honour of Luís Sanchez, professor at Faculdade de Ciências da Universidade de Lisboa, on the occasion of his 70th birthday.

This event counts with the presence of distinguished lecturers who have had a marked importance in Professor Sanchez's scientific career, at both the national and international level, and who are representative of the prolific, productive, and collaborative research environment from which many generations have benefited.

We believe that *Diffsanchez 70* constitutes an opportunity to encourage collaboration among researchers, to promote the exchange of ideas, and to attract young researchers to the field of Differential Equations, bringing together mathematicians, colleagues, and friends.

This booklet compiles all the abstracts of the plenary and invited lectures, as well as the contributed talks presented at *Diffsanchez 70*. The Organising Committee expresses its profound gratitude to all participants, who have contributed by submitting their abstracts to this volume and confirming their presence at this event. Their participation and research are the main contributors for the success of this workshop and for the celebration that it represents.

We also would like to thank the following Universities and Research Centers for their official endorsement and diverse contributions, which include financial and logistical support: Faculdade de Ciências da Universidade de Lisboa, Instituto Superior de Economia e Gestão da Universidade de Lisboa, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa, Instituto Superior de Engenharia de Lisboa, Centro de Matemática, Aplicações Fundamentais e Investigação Operacional and Centro de Matemática Aplicada à Previsão e Decisão Económica.

We also acknowledge Fundação para a Ciência e Tecnologia (FAAC Programme) and Sociedade Portuguesa de Matemática for their institutional and financial support.

We would like to thank those key members of staff who have collaborated in organising this event at different stages, with diverse responsibilities, namely: Andreyna Caires, Inês Oliveira, Rodrigo Marques and Helena Afonso for their secretarial work, and Diogo Neves de Almeida for assuring the digital communication.

We are certain that *Diffsanchez 70* will be a rewarding scientific event which above all will capture and catalyse the outstanding academic, scientific, and human dimension of Luís Sanchez, which has benefited not just us four, but many of his students and others.

The organisers

Ricardo Enguiça
José Maria Gomes
Maria do Rosário Grossinho
Carlota Rebelo

Plenary Lectures

Constant sign solutions of boundary value problems

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In this talk we present several results that ensure the constant sign of the solutions of nonlinear boundary value problems. To this end, we will study the constant sign of the Green's function related to the considered problems. Such constant sign follows from a spectral characterization that avoid to obtain the exact value of the corresponding functions.

Moreover, the use of variational methods allow us to ensure the existence of constan sign solutions for sign changing Green's functions.

The given result are applied to several physical models as, among others, the behavior of a suspension bridge.

Linear stability of a shock profile for a quasilinear Benney system in $\mathbb{R}$

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We study the stability in a partial linearisation framework of a particular travelling wave $(0, \tilde{V})$ of a quasilinear viscous Benney system in $\mathbb{R}$, where $\tilde{V}$ is a standing wave that is shock profile for the conservation law.
This talk is mainly based on a paper in collaboration with Pedro Freitas, published in Nonlinearity in 2018.
The preliminaries are based in two former papers, one in collaboration with M. Figueira and other with M.Figueira and F. Oliveira.

Homoclinics for some singular strong force Lagrangian systems

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We study the existence of solutions homoclinic to the origin for a class of Lagrangian systems with two degrees of freedom of the form

$$\frac{d}{dt}\left(\nabla\Phi(\dot{u}(t))\right) + \nabla_u V(t, u(t)) = 0,$$

where $\Phi\colon \mathbb{R}^2 \to [0, \infty)$ is a G-function in the sense of Trudinger, $V : \mathbb{R} \times \left(\mathbb{R}^2 \setminus \{\xi\}\right) \to (-\infty, 0]$ is a C^1-smooth potential with a single well of infinite depth at a point $\xi \in \mathbb{R}^2 \setminus \{0\}$ and a unique strict global maximum 0 at the origin.

Under a strong force condition around the singular point ξ, depending upon Φ, we prove, via minimization of the action integral and the use of winding number, the existence of at least two geometrically distinct homoclinic solutions $u^{\pm}\colon \mathbb{R} \to \mathbb{R}^2 \setminus \{\xi\}$. This generalizes a result of Rabinowitz for $\ddot{u} + \nabla_u V(t, u) = 0$.

This is a joint work with Marek Izydorek and Joanna Janczewska.

Periodic solutions of the forced Kepler problem

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Co-authors: Alberto Boscaggin and Lei Zhao

Consider the differential equation

$$\ddot{x} = -\frac{x}{|x|^3} + p(t), \quad x \in \mathbb{R}^N$$

in dimension $N = 1, 2$ or 3. The function $p(t)$ is given and periodic. We will present some results on the existence and multiplicity of periodic solutions. In particular we consider solutions with collisions ($x(t_0) = 0$). Some connections with the classical regularization by Levi-Civita will be discussed.

Periodic solutions for nonlinear ODEs with superlinear terms: some topological approaches

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The aim of this talk is to present and discuss some classical and recent results concerning the existence and multiplicity of positive or oscillatory periodic solutions to second order nonlinear ODEs of the form

$$\ddot{x} + f(t, x) = 0 \tag{1}$$

with f having a superlinear growth at infinity. Typical forms of the nonlinearity include terms like $f(t, x) = a(t)g(x)$ or $f(t, x) = a(t)g(x) - e(t)$. The presence of friction terms, leading to

$$\ddot{x} + c\dot{x} + f(t, x) = 0, \tag{2}$$

will be considered as well. Our approach will be mainly focused on the applications of dynamical systems methods concerning fixed points for planar maps and their iterates, such as the Poincaré–Birkhoff fixed point theorem and some of its variants, or functional-analytic methods based on topological degree. For certain special cases of equation (1) or (2) we will also compare some results which can be obtained with the two approaches. The presence of singularities in the x-variable will be briefly discussed, too.

Invited Lectures

Traveling waves for a reaction-diffusion equation with bounded flux.

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We deal with the analysis of traveling waves for the parabolic equation

$$u_t = (a(u, u_x))_x + f(u), \;\; t > 0, \; x \in \mathbf{R},$$

where the reaction term f is a classical Fisher term, $f(u) = ru(1-u), \; r > 0$, or a Fisher-like term. The diffusion term $a : [0,1] \times \mathbf{R} \to \mathbf{R}$ is continuous, bounded and increasing on its second argument.

A phase space analysis provides a value, $\sigma^* := \sigma^*(a, f)$, so that classical profiles of traveling waves moving to speed bigger than σ^* appear.

An analysis by viscosity arguments of the equation

$$u_t = (a(u, u_x))_x + \varepsilon u_{xx} + f(u), \;\; t > 0, \; x \in \mathbf{R},$$

with ε a small parameter, allow us to prove the existence of profiles for the first equation that moves to speed σ^*, and even to speeds lower than σ^*. Those profiles are not classical, but they make sense in the context of entropy solutions.

All the results are part of an upcoming paper in collaboration with Juan Campos.

An Optimal Design Problem under Non-Standard Growth Conditions and a Perimeter Penalization

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In this talk I will present some joint work with E. Zappale where we investigated the possibility of obtaining a measure representation for two functionals arising in the context of optimal design problems, under non-standard growth conditions and a perimeter penalization.

We show that one of the functionals under consideration only admits a weak measure representation, whereas for the other a strong measure representation holds. Under some convexity assumptions, we provide a partial characterisation of the corresponding measures, a full representation is obtained in the one-dimensional setting.

The transition problem for meromorphic polar functions

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In very general conditions, meromorphic polar functions (i.e. functions exhibiting some kind of positive or co-positive definiteness) separate the complex plane into horizontal or vertical strips of holomorphy and polarity, in each of which they are characterized as integral transforms of exponentially finite measures. These measures characterize both the function and the strip.

We study the problem of transition between different holomorphy strips, proving a transition formula which relates the measures on neighbouring strips of polarity. Applications to especially significant examples like the Γ, ζ or Bessel functions are performed.

Pattern formation in Keller-Segel models with flux-saturated mechanisms

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Chemotaxis is the directed movement of cells in response to chemical gradients. It plays an important role in many biological fields, such as embrogenesis, immunology, cancer growth and wound healing. This mechanisms induces many living organisms to locate nutrients, avoid predators or find animals of the same species. The classical Keller-Segel model takes in account the density of a population of cells $u = u(t,x)$ and the concentration of a chemoattractant $S = S(t,x)$. This model reads as

$$u_t = \nabla_x \cdot (D(u, \nabla_x u) - \alpha u \nabla_x S).$$

When $D(u, \nabla_x u) = \nabla_x u$, the classical Keller-Segel model arises, where the chemoattractant verifies

$$\gamma S_t = \Delta_x S + bu.$$

In some sense the cells are atracted by a chemical producted by themselves $(\alpha, \gamma, b > 0)$.

Here we will analize the case were $D(u, \nabla u)$ is bounded on the second argument and therefore we have a nonuniformly elliptic operator. We will make emphasis on the so called Flux-Satured equations.

Existence results for quasilinear elliptic systems

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The aim of this talk is to present some existence results for (p, q)-Laplacian systems. More precisely, the existence of at least one positive solution is studied for a system involving a Dirichlet boundary value conditions, in which the non-linearities dealing with convection terms and are singular with respect to the solution. While, the existence of a nodal solution is ensured for a parametric system with Neumann boundary conditions.

The approach adopted is mainly based on a sub-super solution method, combined with the Schauder fixed point theorem, in the case of a constant sign solution, and with the pseudomonotone operators theory to find a sign-changing solution.

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Periodic oscillations of damped ϕ-Laplacian pendulum equations

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Based on a continuation theorem by Capietto, Mawhin and Zanolin we prove the existence of at least two different T-periodic solutions for the differential equation

$$(\phi(x'))' + k\,x' + a \sin x = e(t), \tag{1}$$

where $\phi :]-A, A[\longrightarrow \mathbb{R}$ is an increasing and odd homeomorphism with $\phi(0) = 0$, $0 < A \leq +\infty$, $k \geq 0$, $a > 0$ and e is a T-periodic function with $\bar{e} := \frac{1}{T}\int_0^T e(t)dt = 0$ for fixed $T > 0$.

Notice that both classical and relativistic pendulum equations, see [5, 7], fit into the scope of (1). In particular we improve or complement some previous results in the literature [1, 3, 4, 7, 8].

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Critical well-posedness for the modified Korteweg-de Vries equation and self-similar dynamics

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We consider the modified Korteweg-de Vries equation over the real line

$$u_t + u_{xxx} = \pm(u^3)_x.$$

This equation arises, for example, in the theory of water waves and vortex filaments in fluid dynamics. A particular class of solutions to (mKdV) are those which do not change under scaling transformations, the so-called *self-similar* solutions. Self-similar solutions blow-up when $t \to 0$ and determine the asymptotic behaviour of the evolution problem at $t = +\infty$.

The known local well-posedness results for the (mKdV) fail when one considers critical spaces, where the norm is scaling-invariant. This also means that self-similar solutions lie outside of the scope of these results. Consequently, the dynamics of (mKdV) around self-similar solutions are currently unknown.

In this talk, we will show existence and uniqueness of solutions to the (mKdV) lying on a critical space which includes both regular and self-similar solutions. Afterwards, we present several results regarding global existence, asymptotic behaviour at $t = +\infty$ and blow-up phenomena at $t = 0$.

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Multiplicity results for a class of asymptotically linear systems of second-order ordinary differential equations

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We study multiplicity of solutions to a Dirichlet problem associated with a planar system of the form

$$\begin{cases} u''(t) + A(t, u(t))u(t) = 0, & t \in [0, \pi], \\ u(0) = u(\pi) = 0, \end{cases}$$

where $A : [0, \pi] \times \mathbb{R}^2 \to GL_s(\mathbb{R}^2)$ is a continuous function satisfying asymptotically linear conditions. The multiplicity result is expressed in term of the Morse indexes of the linearizations at zero and infinity: the gap between the Morse indexes provides a lower estimate on the number of solutions. The proof is developed in the framework of the shooting methods and it is based on the concepts of phase angles.

Asymptotic behaviour for Nicholson systems with patch structure

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We study the global asymptotic behaviour of solutions for a Nicholson's blowflies system with patch structure and multiple discrete delays:

$$x_i'(t) = -d_i(t)x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{m} \beta_{ik}(t)x_i(t-\tau_{ik}(t))e^{-c_i(t)x_i(t-\tau_{ik}(t))},\ i = 1, \dots, n, \qquad (1)$$

where all the coefficient and delay functions are continuous, nonnegative and bounded, $d_i(t) > 0, c_i(t) \geq c_i > 0$ and $\beta_i(t) := \sum_{k=1}^{m} \beta_{ik}(t) > 0$ for $t \geq 0$, $i, j = 1, \dots, n, k = 1, \dots, m$.

For the autonomous version of (1), an overview of results concerning the total or partial extinction of the populations, uniform persistence, existence and absolute global asymptotic stability of a positive equilibrium is presented, see [3, 4]. A criterion for the global attractivity of the positive equilibrium depending on the size of delays is also given [2], extending results in [1]. Most of these results rely on some properties of the so-called community matrix and on the specific shape of the nonlinearity.

For non-autonomous systems (1), sufficient conditions for both the extinction of the populations in all the patches and the permanence of the system were established in [3]. In this case, (1) is treated as a perturbation of the linear homogeneous cooperative ODE system $x_i'(t) = -d_i(t)x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)x_j(t)$ $(1 \leq i \leq n)$, for which conditions for its asymptotic stability are imposed; although the nonlinear terms in (1) are non-monotone, techniques of cooperative DDEs are used.

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On the use of Poincaré-Birkhoff theorem for infinite-dimensional systems

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I will review some recent results on the multiplicity of periodic solutions of Hamiltonian systems, obtained by a higher dimensional version of the Poincar-Birkhoff theorem. When the dimension is infinite, a limit procedure may provide the existence of at least one periodic solution. Different settings will be considered, comparing the available techniques.

Heteroclinic connections for a double well potential with an asymptotically periodic coefficient

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We prove the existence of monotone heteroclinic solutions to a scalar equation of the kind $u'' = a(t)V'(u)$ under the following assumptions: $V \in C^2(\mathbf{R})$ is a non-negative double well potential which admits just one critical point between the two wells, $a(t)$ is measurable, asymptotically periodic and such that $\inf a > 0$, $\sup a < +\infty$. In particular, we improve earlier results in the so called asymptotically autonomous case, when the periodic part of a, say $\tilde{a}$, is constant, i.e. $a(t)$ converges to a positive value l as $|t| \to +\infty$. Furthermore, whenever $\tilde{a}$ fulfils a suitable non-degeneracy condition, the solutions are shown to be infinitely many.

Some recent results on traveling waves for reaction-diffusion models with saturation

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We present some results for the one-dimensional reaction-diffusion equation

$$u_t = \epsilon \left(\frac{u_x}{\sqrt{1+u_x^2}} \right)_x + f(u), \quad u = u(x,t),\ x \in \mathbf{R},\ t \in \mathbf{R},$$

where the diffusion is of mean curvature type (a particular case of *strongly saturating diffusion*) and the reaction term f can be monostable or bistable. We deal with the existence of heteroclinic traveling wave-type solutions and with their convergence properties for $\epsilon \to 0$. The exploited technique relies on a suitable change of variables reducing the order of the problem, together with a direct convergence analysis making use of the properties of the mean curvature operator. Part of the discussion is based on previous works in collaboration with L. Sanchez and M. Strani.

Solutions of the wave equation bounded at the Big Bang

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By solving a singular initial value problem, we prove the existence of solutions of the wave equation $\Box_g \phi = 0$ which are bounded at the Big Bang in the Friedmann-Lemaître-Robertson-Walker cosmological models. More precisely, we show that given any function $A \in \mathbb{H}^3(\Sigma)$ (where $\Sigma = \mathbb{R}^n, \mathbb{S}^n$ or $\mathbb{H}^n$ models the spatial hypersurfaces) there exists a unique solution ϕ of the wave equation converging to A in $H^1(\Sigma)$ at the Big Bang, and whose time derivative is suitably controlled in $L^2(\Sigma)$. This is joint work with José Natário and Jorge Silva.

Existence and multiplicity of solutions for second order problems with different boundary conditions

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The talk concerns two different nonlinear second order problems. The existence of positive solutions for nonlinear second order singular equations, with not variational structure, will be presented analysing how the singularity influences the presence of a Dirichlet type solution in alternative to a homoclinic one. Some further attention will be reserved for the so-called periodic solution of Dirichlet type. This is a joint paper with P. Candito, L. Sanchez and M. Zamora.
The multiplicity of periodic solutions will be also presented, via a variational approach, for the study of a second order Hamiltonian system depending on a parameter.

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Delayed gamma models: the Lasota and Mackey-Glass equations revisited

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We consider the family of delay-differential equations $x'(t) = -\alpha x(t) + x^{\gamma}(t-\tau)\, F(x(t-\tau))$, where $\alpha, \tau > 0$, $\gamma \geq 0$, and $F : [0,\infty) \to (0,\infty)$ is a smooth decreasing function.

As far as we know, examples of equations of the form () start with two papers published in 1977. Lasota [2] and Mackey and Glass [4] proposed two similar models for the blood cell production in terms of delay differential equations. On the one hand, the equation proposed by Lasota in [2] is

$$x'(t) = -\alpha x(t) + \beta x^{\gamma}(t-\tau) e^{-\delta x(t-\tau)}\,, \tag{1}$$

with $\alpha, \beta, \delta, \tau > 0, \gamma \geq 0$. On the other hand, Mackey and Glass proposed two equations for hematopoiesis, that can be written in the form

$$x'(t) = -\alpha x(t) + \frac{\beta x^{\gamma}(t-\tau)}{1 + x^{m}(t-\tau)},\ \text{with } \alpha, \beta, m > 0,\ \gamma = 0 \text{ or } \gamma = 1. \tag{2}$$

Equation (1) with different values of γ has been used in different mathematical models. The limit case $\gamma = 0$ was proposed by Lasota and Wazewska in 1976 to model erythropoiesis, while the general case $\gamma > 0$ was introduced later by Lasota [2] to model disturbed erythropoiesis; for $0 < \gamma < 1$, (1) is a generalization of the fundamental Solow's neoclassical growth model in economics [1]; when $\gamma = 1$, equation (1) is the famous Nicholson's blowflies equation. For $\gamma > 1$, (1) and (2) have been proposed to study populations subject to Allee effects in [3] and [5], respectively.

We present some new results on the global dynamics of () in the cases $0 < \gamma < 1$ and $\gamma > 1$, paying special attention to permanence and stability. Bifurcation diagrams using relevant model parameters show some interesting features, such as stability switches and extinction windows due to sudden collapses. In the framework of population dynamics, it is especially interesting to study the influence of γ, which can be considered as a cooperation parameter.

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On a sharp measure control condition and dynamics of semilinear wave equations

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This talk is concerned with dynamics of wave equations with locally distributed damping. We present a characterization of sharp control/damping regions in the sense that summed interior and boundary measures is arbitrarily small. The construction of this class of open sets is purely geometric and allow us to prove a new observability inequality and also a unique continuation property. If time permits, we will discuss the existence of finite dimensional global attractors for a critical wave equation under this new sharp measure control condition.

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Positive solutions to a singular quasi-linear elliptic system in $\mathbb{R}^N$

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The existence of positive solutions $(u,v) \in \mathcal{D}^{1,p_1}(\mathbb{R}^N) \times \mathcal{D}^{1,p_2}(\mathbb{R}^N)$ to the system of quasilinear elliptic equations

$$\begin{cases} -\Delta_{p_1} u = a_1(x) f(u,v) & \text{in } \mathbb{R}^N, \\ -\Delta_{p_2} v = a_2(x) g(u,v) & \text{in } \mathbb{R}^N, \end{cases} \tag{P}$$

where $N \geq 3$, $1 < p_i < N$, while Δ_{p_i} denotes the p_i-Laplace differential operator, is established [1] via suitable a priori estimates and Schauder's fixed point theorem. Nonlinearities $f, g : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ are continuous and fulfill the condition

($\mathrm{H}_{f,g}$) There exist $m_i, M_i > 0$, $i = 1, 2$, such that

$$m_1 s^{\alpha_1} \leq f(s,t) \leq M_1 s^{\alpha_1}(1 + t^{\beta_1}), \quad m_2 t^{\beta_2} \leq g(s,t) \leq M_2(1 + s^{\alpha_2}) t^{\beta_2}$$

for all $s, t \in \mathbb{R}^+$, with $-1 < \alpha_1, \beta_2 < 0 < \alpha_2, \beta_1$,

$$\alpha_1 + \alpha_2 < p_1 - 1, \quad \beta_1 + \beta_2 < p_2 - 1, \tag{0.1}$$

as well as

$$\beta_1 < \frac{p_2^*}{p_1^*} \min\{p_1 - 1, p_1^* - p_1\}, \quad \alpha_2 < \frac{p_1^*}{p_2^*} \min\{p_2 - 1, p_2^* - p_2\}.$$

Here, p_i^* denotes the critical Sobolev exponent corresponding to p_i, namely $p_i^* := \frac{Np_i}{N - p_i}$. Coefficients $a_i : \mathbb{R}^N \to \mathbb{R}$ satisfy the assumption

(H_a) $a_i(x) > 0$ a.e. in $\mathbb{R}^N$ and $a_i \in L^1(\mathbb{R}^N) \cap L^{\zeta_i}(\mathbb{R}^N)$, where $\displaystyle\frac{1}{\zeta_1} \leq 1 - \frac{p_1}{p_1^*} - \frac{\beta_1}{p_2^*}$, $\displaystyle\frac{1}{\zeta_2} \leq 1 - \frac{p_2}{p_2^*} - \frac{\alpha_2}{p_1^*}$.

The most interesting aspect of (P) probably lies in the fact that both f and g can exhibit singularities through $\mathbb{R}^N$, which, without loss of generality, are located at zero. Indeed, $-1 < \alpha_1, \beta_2 < 0$ by ($\mathrm{H}_{f,g}$). It represents a serious difficulty to overcome and is rarely handled in the literature: variational methods do not work, at least in a direct way, because the Euler functional associated with (P) is not well defined; a similar comment holds for sub-super-solution techniques, usually employed in the case of bounded domains. Hence, one is naturally led to apply fixed point results.

As far as I know, singular systems in the whole space have been investigated only for $p := q := 2$, essentially exploiting the linearity of involved differential operators. The situation looks quite different when a bounded domain replaces $\mathbb{R}^N$. In such a case, many singular systems fitting the framework of (P) have been studied; see the bibliography of [1].

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From linear drag to Poynting-Robertson drag and beyond: strenghtening the singularity in a dissipative Kepler Problem

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In this talk we consider the Kepler problem with a family of dissipations of the form $-k\frac{\dot{x}}{|x|^\beta}, k > 0$, $\beta \geq 0$. We present some results about the qualitative dynamics as β increases from zero (linear drag, studied in [1],[2]) to infinity. In particular, in [3] we detect some threshold values of β for which qualitative changes in the global dynamics occur. In the case $\beta = 2$ we refine some previous results and prove that, unlike for the case of the linear drag, the asymptotic Runge-Lenz vector is discontinuous.

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A chromaticity-brightness model for color images denoising

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In this talk a variational model for imaging segmentation and denoising color images is proposed. The model combines Meyer's "u+v" decomposition with a chromaticity-brightness framework and is expressed by a minimization of energy integral functionals depending on a small parameter $\epsilon > 0$. The asymptotic behavior as $\epsilon \to 0^+$ is characterized, and convergence of infima, almost minimizers, and energies are established. In particular, an integral representation of the lower semicontinuous envelope, with respect to the L^1-norm, of functionals with linear growth and defined for maps taking values on a certain compact manifold is provided. This study escapes the realm of previous results since the underlying manifold has boundary, and the integrand and its recession function fail to satisfy hypotheses commonly assumed in the literature. The main tools are Γ-convergence and relaxation techniques.

Functional coupled systems and applications

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This talk is concerned with the existence of solutions for first order fully coupled systems with coupled functional boundary conditions, which generalize the usual boundary assumptions and may be applied to most of the classical cases.

The arguments used are based on the Arzèla Ascoli theorem and Schauder's fixed point theorem.

Two applications will be considered: a mathematical model of the thyroid-pituitary interaction and their homeostatic mechanism and a SIRS model.

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On some parabolic Liouville theorems

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A Liouville theorem for a PDE provides growth/sign conditions ensuring triviality of the corresponding solutions. For the heat equation on a non-compact Riemannian manifold a fruitful setting is the one of nonnegative *ancient solutions* i.e. solutions defined on $(-\infty, T) \times M$. Liouville theorems in this framework are usually obtained through refined differential Harnack inequalities such as Li-Yau's [1] or Hamilton's one, (the non-compact case of the latter beign treated in Souplet-Zhang [3]). We present a simplified functional analytic proof which also gives the optimal growth conditions in the relevant Liouville theorem, which then reads as follows.

Theorem 1 *Let M be a complete non-compact manifold with $\mathrm{Ric} \geq 0$ and u be a non-negative solution to $u_t = \Delta u$ in $(-\infty, T) \times M$. If for some time $t_0 < T$ it holds $u(x, t_0) \leq e^{o(d(x))}$ as $d(x) := \mathrm{dist}(x, x_0) \to +\infty$, then u is constant.*

Possible extensions to weaker curvature bounds such as $\mathrm{Ric} \geq -k^2$ will be discussed, motivated by the richer variety of phenomena which appear in the model case of the hyperbolic space.

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Periodic solutions of Nonlinear Differential Equations

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The existence of solutions of singular ordinary differential equations is an important question. In this area the contribution of L. Sanchez is very relevant; see for example [1, 2].

For given nonlinear differential equations it may occur that there are no periodic solutions. By introducing impulses at prescribed instants, periodic solutions may appear [3].

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Characterizing the formation of singularities in a quasilinear indefinite Neumann problem

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We discuss the regularity properties of the positive bounded variation solutions of the quasilinear indefinite Neumann problem

$$-(u'/\sqrt{1+u'^2})' = \lambda a(x) f(u) \ \text{ in } (0,1), \quad u'(0)=0,\ u'(1)=0. \tag{1}$$

Here, $\lambda \in \mathbb{R}$ is a positive parameter, $a \in L^\infty(0,1)$ changes sign once in $(0,1)$, $f \in C^1(\mathbb{R})$ is positive and increasing in $(0,+\infty)$, with a potential F superlinear at $+\infty$.
We first provide a precise description of the asymptotic profile of the solutions of (1) and then we prove that the appearance of singular solutions is characterized by the integrability of the profile, which is in turn equivalent to requiring a Keller-Osserman condition on the weight a at its nodal point.
This talk reports on a joint research with Julian López-Gómez (Universidad Complutense de Madrid).

Some results on the existence of constant-sign solutions for a class of second-order functional differential equations

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Functional differential equations are an important tool to model phenomena with memory effects (for instance, see [3]). In this work, we consider some properties of the solutions to a class of second-order functional differential equations with piecewise constant arguments. Inspired by the works in [2, 4], where some related first-order periodic boundary value problems were studied, we considered in [5] the existence of solutions for second-order functional differential equations of the type

$$\left\{ \begin{array}{c} x''(t) + ax'(t) + bx(t) + cx'([t]) + dx([t]) = \sigma(t),\ t \in J = [0, T], \\ x(0) = x(T), \quad x'(0) = x'(T) + \lambda, \end{array} \right. \tag{1}$$

by analyzing the existence of solutions for the impulsive periodic boundary value problems

$$\left\{ \begin{array}{c} x''(t) + ax'(t) + bx(t) + cx'([t]) + dx([t]) = 0, \qquad t \in \mathbb{R}, \\ x(0) = x(T), \quad x'(0^-) = x'(T^+), \quad x'(s^+) = x'(s^-) + 1, \end{array} \right. \tag{2}$$

where $s \in J$, a, b, c, d, $\lambda \in \mathbb{R}$ and $T > 0$. We also refer to [6] for some extensions to variable coefficients.

Here, we study some of the properties of the solutions to (1), defined in terms of the kernel derived from the solutions to (2). In particular, we provide some sufficient conditions for the uniqueness of a constant-sign solution to (1) by analyzing the properties of its Green's function. Due to the tedious expressions obtained for the kernels, we explore some procedures that allow to give sufficient conditions for the existence of solutions with a constant sign for the linear equation, and we show some of the implications of these results in the study of other more general problems. Several results in this work are part of a joint work with Sebastián Buedo-Fernández, Daniel Cao Labora and Stepan A. Tersian [1].

References

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Cardiovascular mathematics with applications to some clinical studies

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Mathematical modeling and simulations of the human circulatory system is a challenging and complex wide-range multidisciplinary research field that has seen a tremendous growth in the last few years. This field, with a strong socio-economic impact, is rapidly progressing motivated by the fact that cardiovascular diseases are a major cause of death in developed countries.

In this talk we will consider some mathematical models and simulations of the cardiovascular system and comment on their significance to yield realistic and accurate numerical results, using stable, reliable and efficient computational methods.

Results on the simulation of some image-based patient-specific clinical cases will also be presented.

Sharp concentration estimates near criticality for radial sign-changing solutions of Dirichlet and Neumann problems

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We consider radial solutions of the slightly subcritical problem $-\Delta u_\varepsilon = |u_\varepsilon|^{\frac{4}{n-2}-\varepsilon} u_\varepsilon$ either on $\mathbb{R}^n$ ($n \geq 3$) or in a ball B satisfying Dirichlet or Neumann boundary conditions. In particular, we provide sharp rates and constants describing the asymptotic behavior (as $\varepsilon \to 0$) of all local minima and maxima of u_ε as well as its derivative at roots. Our proof is done by induction and uses energy estimates, blow-up/normalization techniques, a radial pointwise Pohozaev identity, and some ODE arguments. As corollaries, we complement a known asymptotic approximation of the Dirichlet nodal solution in terms of a tower of bubbles and present a similar formula for the Neumann problem.

References

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Subharmonic solutions of weakly coupled Hamiltonian systems

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We prove the existence and multiplicity of subharmonic solutions for Hamiltonian systems obtained as perturbations of N planar uncoupled systems. The proof is carried out by the use of a generalized version of the Poincaré-Birkhoff Theorem. The results were obtained together with A. Fonda (University of Trieste).

Recurrence relations with reflection

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In this talk we develop an algebraic theory of linear recurrence equations and systems with constant coefficients and reflection. We obtain explicit solutions and the Green's functions associated to different problems under general linear boundary conditions. Furthermore, we establish different relations connecting the algebras of recurrence and differential operators, showing the similarities and differences between them.

References

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On the curvature and torsion effects in thin waveguides

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We consider Schrödinger's equation in a domain with an infinite potential at the boundary. For a curved tube or for a thin shell, the wave function turns out to be a solution of an eigenvalue problem for Laplace's operator.

In this work, we study the limit problem as the diameter of the tube's cross section or the thickness of the shell goes to zero and show the effect of the curvature and of the torsion functions on the energy levels (eigenvalues) and on the wave functions (eigenvectors).

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International Workshop on Differential Equations
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Opial type results for second order measure equations

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The contribution is based on the joint research with **Giselle Antunes Monteiro** and **Antonín Slavík** (cf. [1]). First, we will recall some of the basic properties of the Kurzweil-Stieltjes integral and our recent contributions concerning the continuous dependence on a parameter k of solutions to linear integral equations of the form

$$x(t) = \widetilde{x}_k + \int_a^t \mathrm{d}[A_k]\, x + f_k(t) - f_k(a)\,, \quad t \in [a,b\,]\,, k \in \mathbb{N}\,,$$

where $-\infty < a < b < \infty$, X is a Banach space, $L(X)$ is the Banach space of linear bounded operators on X, $\widetilde{x}_k \in X$, $A_k : [a,b\,] \to L(X)$ have bounded variations on $[a,b\,]$, $f_k : [a,b\,] \to X$ are regulated on $[a,b\,]$ and the integrals are understood as the abstract Kurzweil-Stieltjes integrals.

As a consequence, we will extend Opial type results to second order measure equations as well as to dynamic equations on time scales.

References

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Integral representation of local energy functionals on $BD(\Omega)$

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Several phenomena in phase transition, fracture mechanics, liquid crystals, can be modelled as energy minimization problems where the natural energy shows volume and surface terms and possibly more complex low-order energy terms. In many cases the energy functional is obtained as a limit of approximating functionals and some of its properties can be deduced from the approximation process. In view of these purposes it is a crucial step to obtain an integral representation of the energy.

In this talk I will present a result on the integral representation of local functionals defined on the space BD of functions of bounded deformations.

This topic is an example of the scientific activity produced at CMAF research center in the last decades, and in particular during the period were the center was lead by L. Sanchez. Indeed the first paper on this topic [BFM98] and the two following [BFT00, ET01] were all co-authored by CMAF researchers.

The relatively long time between our result and those is due to the issue of characterizing the Cantor part of BD functions. This has been done only recently in [DPR16] and thus made the proof of our result a doable extension of previous techniques.

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Existence and stability results for the Liebau-type equation

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The talk is based on the join research with José Ángel Cid, Shenjung Li, and Feng Wang ([1], [2]). We will discuss existence, localization and stability of positive solutions for the equation

$$x'' + ax' = r(t)x^{\alpha} - s(t)x^{\beta}$$

subject to periodic boundary conditions $x(0) = x(T)$, $x'(0) = x'(T)$. The main tools are the method of upper and lower solutions and the averaging method. As an application, we present sufficient conditions for the existence and stability of periodic solutions for the Liebau-type differential equation

$$u'' + au' = \frac{1}{u}(e(t) - b(u')^2) - c.$$

References

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Contributed Talks

Why should Cauchy-Lindelöf-Lipschitz-Picard Theorem hold?

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We give an intuitive proof for the well-known Cauchy-Lindelöf-Lipschitz-Picard Theorem, using shorter and different arguments to the standard ones. If we consider an ordinary differential equation $x' = f(t, x)$ and we denote the point of interest by $p = (t_0, x_0)$, this result roughly states that the existence and local uniqueness of solution for the previous ODE at p is ensured, provided that f is continuous at p and locally Lipschitz with respect to the second variable at p. In this talk, we will show why the multiplicity of solution at p is intuitively non-compatible with the hypotheses in Cauchy-Lindelöf-Lipschitz-Picard Theorem, since their coexistence gives lead to an apparent geometric contradiction. This contradiction is easily formalised and it is a direct consequence of the fact that $g(t) \neq O(g'(t))$ at $t = t_0$ for any non-zero continuously differentiable function such that $g(t_0) = g'(t_0) = 0$, where O denotes the Bachmann-Landau notation. The content of the talk is mainly contained in [1].

References

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Travelling wave profiles in some models with nonlinear diffusion

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We present versions of some classical results on the speed of travelling waves for FKPP equations considering different models of nonlinear diffusion.
More precisely, we will discuss existence and some properties of the monotone solutions of the boundary value problem

$$\begin{aligned}(P(u'))' - cu' + f(u) = 0,\\ u(-\infty) = 0,\ u(+\infty) = 1,\end{aligned}$$

which is motivated by the model case with $P(x) = \frac{x}{\sqrt{1-x^2}}$. This in turn corresponds to the problem of finding travelling waves for an analogue of the FKPP equation in the context of the Minkowski curvature operator. The parameter c is the wave speed and we assume that f is a continuous function in $[0, 1]$ such that $f(0) = f(1) = 0$.
We obtain a theory of admissible speeds and some other properties that generalise classical and recent results. The talk is inspired in the paper [1].

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Existence and localization of positive periodic solutions for some time-dependent predator-prey systems via fixed point theory

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The classical predator-prey model can be written as

$$\begin{cases} x'(t) = a\,x(t) - \lambda\,x(t)\,y(t), \\ y'(t) = -b\,y(t) + c\,\lambda\,x(t)\,y(t), \end{cases} \tag{1}$$

for every $t \in \mathbb{R}$, where a, b, c and λ are positive. There exist in the literature several papers devoted to the study of many generalizations of this system and we were specially inspired by two of them.
In 1996, Tsvetkov considered a natural generalization of system (1) allowing the positive numbers to be non-negative ω-periodic functions. The assumption was justified by the periodicity of factors like seasonal effects of weather, harvesting or hunting seasons, etc. By means of some fixed point index results, he was able to prove the existence of positive ω-periodic solutions. Another approach was recently carried out in [2], where the prey growth was assumed to be logistic instead of linear and, the attack rate of the predator population λ was modified to include hunting cooperation between predators.
Attracted by the models studied in [2] and the approach developed in [3], we decided to study the existence of positive ω-periodic solutions for predator-prey systems of the form:

$$\begin{cases} x'(t) = a(t)\,x(t)\left(1 - \frac{x(t)}{K}\right) - \varphi(t, x(t), y(t))\,x(t)\,y(t), \\ y'(t) = -b(t)\,y(t) + c(t)\,\varphi(t, x(t), y(t))\,x(t)\,y(t), \end{cases}$$

for every $t \in [0, \omega]$, where a, b, c, $\varphi(\cdot, x, y)$ are ω-periodic functions and $K \in (0, +\infty]$ (linear growth: $K = +\infty$). At the beginning we tried to apply the classical and vector versions of Krasnosel'skii compression-expansion fixed point theorem (see [1]), but we do not succeed. Fortunately, there is an homotopy version of the classical expansion result that can be applied.
By means of this homotopy version, we prove the existence of ω-periodic orbits when φ fulfil some suitable conditions. Then, we considered some particular expressions for φ inspired by [2] and show that they satisfy the required conditions. In particular, we revisit the models considered by Tsvetkov and M. Teixeira Alves & F. M. Hilker.
It is also important to be mentioned that we can be localizing steady states of the system, since they are ω-periodic functions for every $\omega \geq 0$. Therefore, we state sufficient conditions to ensure we localize non-trivial periodic orbits.

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Vector Cross Product Differential Equations in $\mathbb{R}^3$ and in $\mathbb{R}^7$

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Given a d-dimensional vector space V be over a field F, equipped with a nondegenerate symmetric bilinear form $\langle \cdot, \cdot \rangle$, a bilinear map $\times : V \times V \to V$ is a 2-*fold vector cross product* (vcp) if

$$1.\ \langle u \times v, u \rangle = \langle u \times v, v \rangle = 0 \qquad\qquad 2.\ \langle u \times v, u \times v \rangle = \begin{vmatrix} \langle u, u \rangle & \langle u, v \rangle \\ \langle v, u \rangle & \langle v, v \rangle \end{vmatrix}.$$

This is a generalization of the usual vcp in $\mathbb{R}^3$. The values of n for which the Euclidean spaces $\mathbb{R}^n$ can be equipped with a 2-fold vcp are restricted to 1 (trivial case), 3 and 7. The 2-fold vcp can be found in mathematical models of physical processes, control theory problems in particular, which involve differential equations. In [3], [4], via certain 3×3 skewsymmetric matrices, it is used in the description of spacecraft attitude control. The analogue problem in the 7-dimensional case is also studied.

If $u \in \mathbb{R}^7$ (respectively, $\mathbb{R}^3$), then S_u is the skewsymmetric matrix in $\mathbb{R}^{7\times 7}$ (resp., $\mathbb{R}^{3\times 3}$) such that:

$$S_u x = u \times x \tag{1}$$

for any $x \in \mathbb{R}^7$ (resp., $\mathbb{R}^3$). Thus, given $u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \end{bmatrix}^T$ (resp., $\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$),

$$S_u = \left[\begin{array}{c|c} E & F \\ \hline G & H \end{array}\right] = \left[\begin{array}{rrr|rrrr} 0 & -u_3 & u_2 & -u_5 & u_4 & -u_7 & u_6 \\ u_3 & 0 & -u_1 & -u_6 & u_7 & u_4 & -u_5 \\ -u_2 & u_1 & 0 & u_7 & u_6 & -u_5 & -u_4 \\ \hline u_5 & u_6 & -u_7 & 0 & -u_1 & -u_2 & u_3 \\ -u_4 & -u_7 & -u_6 & u_1 & 0 & u_3 & u_2 \\ u_7 & -u_4 & u_5 & u_2 & -u_3 & 0 & -u_1 \\ -u_6 & u_5 & u_4 & -u_3 & -u_2 & u_1 & 0 \end{array}\right] \text{(resp., } E\text{)}.$$

Let $n \in \{3, 7\}$, $a,\ b \in \mathbb{R}^n \backslash \{0\}$, $\alpha \in \mathbb{R} \backslash \{0\}$, $f = f(t)$ a $\mathbb{R}^n$-valued function of the real variable t, continuous in some interval containing t_0, and $x = x(t)$ an unknown $\mathbb{R}^n$-valued function of the real variable t, with initial condition $x(t_0) = x_0$. In this talk, we give explicit solutions of the following vcp differential equations:

$$\dot{x} + b \times x = 0, \tag{2}$$
$$\dot{x} + b \times x = f, \tag{3}$$
$$a \times \dot{x} + b \times x + \alpha x = 0 \tag{4}$$

where in (4) $x_0 \in \mathbb{R}^n$ is a consistent initial vector. In order to solve these equations we use (1), and applying either the classical theory or convenient Drazin inverses of elements belonging to the class of index 1 matrices [2]. This talk is based on a partial version of [1].

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Positive solutions for second-order boundary-value problems with sign changing Green's functions

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We analyze some possibilities of finding positive solutions for second-order nonlinear boundary-value problems related to Hill's equation

$$u''(t) + a(t)\,u(t) = f(t, u(t)),\ t \in [0, T],$$

with the Dirichlet and periodic boundary conditions, for which the corresponding Green's functions change sign.

The obtained results can also be adapted to Neumann and mixed boundary conditions.

These results can be found in [1].

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Positive solutions of a superlinear indefinite prescribed mean curvature problem

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We discuss some recent results concerning the existence of positive solutions of a superlinear indefinite prescribed mean curvature problem

$$\begin{cases} -\mathrm{div}\left(\dfrac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = \lambda a(x) u^p + \mu u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with a regular boundary $\partial\Omega$, $p > 1$ is an exponent satisfying $p < \frac{N+2}{N-2}$ if $N \geq 3$, $\lambda > 0$ and $\mu \leq 0$ are parameters, a is a continuous sign-changing function.

These equations arise in several applications, e.g., to describe phenomena of capillarity for incompressible or compressible fluids as well as phenomena of flux-limited diffusion (cf., [2, 3]).

We exploit a perturbation approach to a limiting semilinear problem and use fixed point index theory in order to prove the existence of positive strong solutions when the parameter λ is large enough. Moreover, we prove that in the $(\lambda, \|u\|_\infty)$-plane there exists a closed connected component of solutions pairs unbounded in λ (cf. [4]). This investigation extends the result in [1] achieved in the case where a is positive. We also show the appearance of bounded variation solutions for λ in some open interval and discuss open questions about the λ-intervals of existence resulting from the two different regularity conditions of the solutions.

This talk is based on joint work with Pierpaolo Omari (University of Trieste).

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First order differential systems with a nonlinear boundary condition via the method of solution-regions

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In recent works of Frigon [1] and Tojo [2], the concept of solution-region was studied for the first time in order to obtain results concerning the existence and multiplicity of solutions of systems of differential equations. In this work we prove some results concerning the concept of admissible region and we consider systems of first order differential equations with nonlinear boundary conditions, improving the works [1, 2] in several ways.

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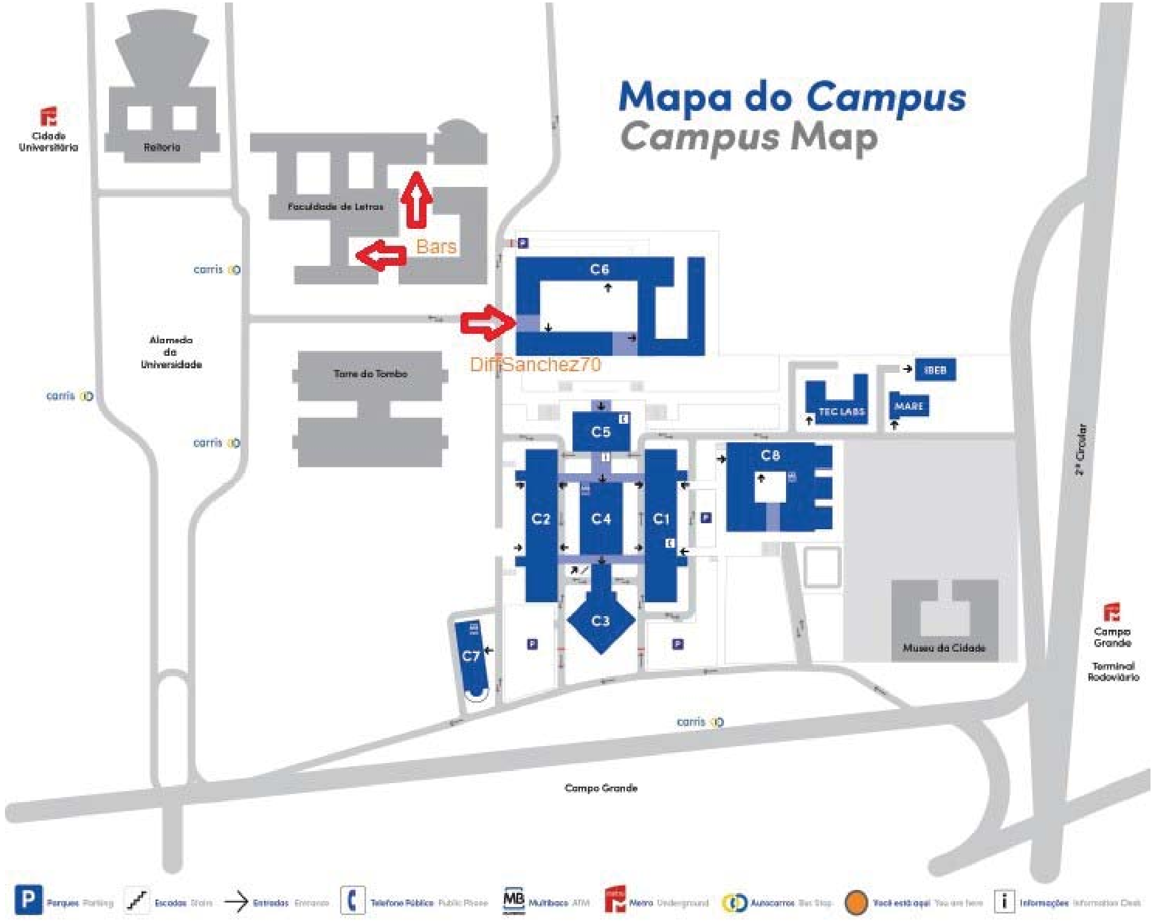
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